Written Exam at the Department of Economics summer 2021

Advanced Game Theory

Final Exam

17 August 2021

(3-hour closed book exam)

Answers only in English.

This exam question consists of 3 pages in total incl. this one

Advanced Game Theory

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Exam 2

The exam is 3 hours. Please answer as many questions as you can. Answer shortly but justify your answers and explain accurately what you are doing. If you are confused about some question statement, please explain clearly what you assume when answering.

1. State whether the statements are TRUE or FALSE. Justify your answer.

   (a) The set of correlated equilibria is the same as the set of Nash equilibria in the following game:

	L	R
T	4, 3	3, 1
M	8, 2	0, 0
B	5, 6	3, 7

   (b) Consider infinitely repeated finite normal form stage game. If a feasible payoff vector v is such that each of its component is strictly larger than the smallest stage game Nash equilibrium payoff for that player, then there exists a $\delta \in (0,1)$ such that v is the payoff vector for some subgame perfect equilibrium when players discount with factor δ.

   (c) Asymmetric information may help to achieve a more efficient equilibrium (= better for all players) in some dynamic games.

   (d) There exists a belief μ such that $((OR, N, AC), \mu)$ is a sequential equilibrium in the following game:
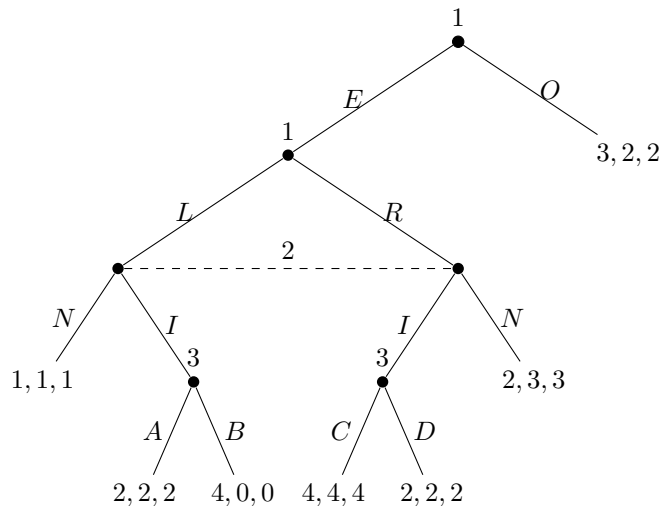
2. This question considers the following game:

	L	R
U	9, 1	0, 0
D	θ, θ	1, 9

(a) Find the unique Nash equilibrium of the game when $\theta = 10$.

(b) Consider a situation where θ is 5 with probability 0.5 and 15 with probability 0.5. P2 does not know the true value of θ but P1 does. Find the unique pure strategy Bayes Nash equilibrium of the game.

(c) Next, consider a situation where the information structure is otherwise as in (b) but P2 receives a signal $s \in \{h, l\}$ of θ: $Pr(s = h | \theta = 5) = 0$ and $Pr(s = h | \theta = 15) = 0.4$. Write the game as a game of incomplete information. Remember to include types for each player.

(d) Show that the following two strategy profiles are Bayes Nash equilibria of the game in part (c): i) P1: D always, P2: R if $s = l$ and L if $s = h$; ii) P1: U if $\theta = 5$ and D if $\theta = 15$, P2: L always.

3. Two firms are trying to force each other out of business and have started a price war. In each period $t = 0, 1, 2, ...$, the firms simultaneously choose between fighting (F) and quitting (Q). Quitting is irreversible and yields payoff 0 from that period onward. If both firms fight, they get payoff $-c$ per period. The game ends as soon as one of the firms quit and then the other firm receives a monopoly profit p per period from that period onward (including the period when the other firm quits). The players discount their costs and profits with discount factor δ. Assume that the firms are not liquidity constrained and can keep fighting forever.

(a) Consider a strategy profile $s_1(t) = F$ for all t and $s_2(t) = Q$ for all t. Is the profile a subgame perfect equilibrium?

(b) Find a subgame perfect equilibrium where both players stop with the same constant probability in each period.

(c) Now, consider the same game but when Firm 2 observes the action taken by Firm 1 before taking its own action. Construct a subgame perfect equilibrium where Firm 1 randomizes in every period and Firm 2 randomizes if Firm 1 fights. Which firm gets a larger expected payoff in that equilibrium?

4. Consider the following (fictional) situation. A housing cooperative (*andelsboligforening*) has 9 identical apartments but only 3 parking slots that are allocated among the shareholders (=apartment owners) based on how long they have lived in the housing cooperative. Now, a new shareholder suggests that the housing cooperative increases the

rent of the parking slots. The revenue from the rents is used to cover general expenses in the housing cooperative so that they benefit all shareholders equally. Does the shareholders' meeting approve the suggestion? The suggestion gets approved if a majority of the shareholders votes for it.

You are allowed to combine sub questions but then you need to state clearly which sub questions you are answering together.

(a) Define a game that describes the situation.

(b) Point out what assumptions you have made in part (a).

(c) What would be a suitable solution concept to solve the game in part (a)? Argue why.

(d) Write down the equations that characterize a solution (this means that a strategy profile that satisfies all of them is a solution).

(e) Either solve the game OR discuss what you would expect to happen in the game (the latter means writing a few sentences where you describe the main tradeoff).

(f) Interpret your results (write a few sentences).